

NONLINEAR OPTICS ■ 20 January 2026

Interband Berry Phase in Laser-Driven Crystals

HHG as an internal interferometer in solids

EREN ERBERK ERKUL

nature.com/articles/s41586-023-06828-5



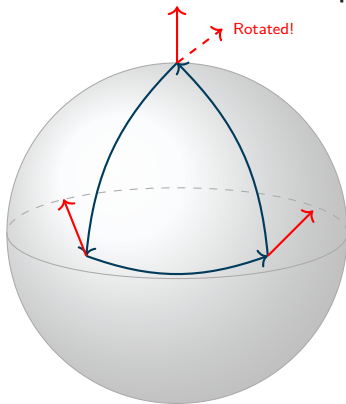
The Origin: Curvature



Carl Friedrich Gauss

Geometric Phase Intuition

Parallel Transport on a Sphere



$$\gamma = \oint \mathcal{A}(\mathbf{R}) \cdot d\mathbf{R}$$

Phase from path geometry

The Quantum Connection



Sir Michael Berry (1984)

Berry phase

$$\gamma = \oint \mathcal{A}(\mathbf{R}) \cdot d\mathbf{R}$$

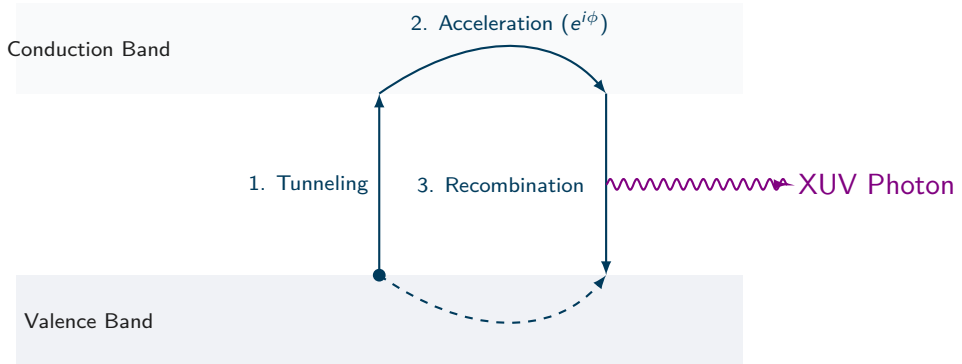
phase from the path geometry

The Core Concept

- HHG → **Internal Interferometer**
- Target: **Interband Geometric Phase**
 - Phase → Interference Spectrum

Mechanism

Time = Loop Size in Reciprocal Space

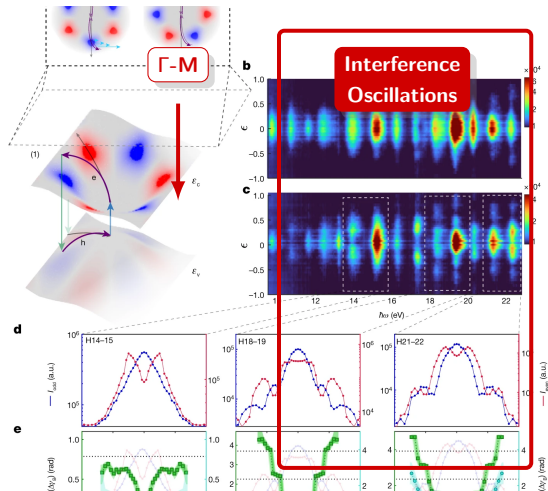


The Measured Phase

$$\gamma_{\text{int}} = \int (\mathcal{A}_c - \mathcal{A}_v) \cdot d\mathbf{k} - \Delta\phi_d$$

intraband connection + dipole-phase jump

Ellipticity bends the loop

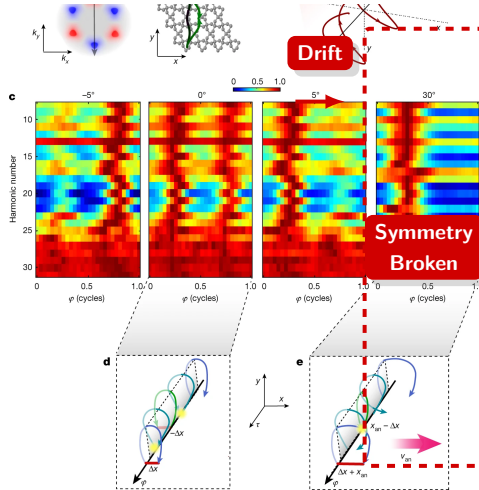


Curvature $\rightarrow$ Drift

$$\dot{\mathbf{r}} = \nabla_{\mathbf{k}} \varepsilon_n(\mathbf{k}) - \dot{\mathbf{k}} \times \boldsymbol{\Omega}_n(\mathbf{k})$$

anomalous velocity controls recombination overlap

Control field reveals curvature

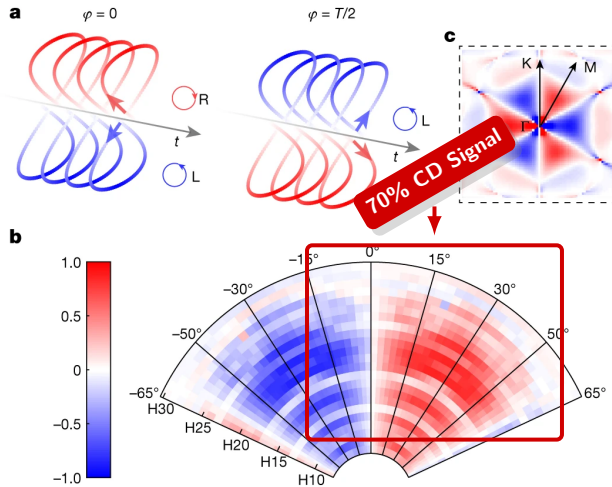


Circular dichroism

$$\text{CD}(\omega) = \frac{I_+(\omega) - I_-(\omega)}{I_+(\omega) + I_-(\omega)}$$

helicity dependence as a geometric fingerprint

Chiral response in HHG



Take-home

- HHG as sub-cycle interferometer
- Resolves interband geometric phase
- Curvature $\rightarrow$ Drift & Dichroism

Questions