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HOLOGRAPHIC SPECTRAL ALIGNMENT

Recovering orientation from the boundary

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THE TAKE-HOME

A 3-D ROTATION → A 1-D SHIFT

NEW ALGORITHM

2-5° MEDIAN ERROR

8x FASTER

THE PROBLEM

Recover R ∈ SO(3) that aligns an observed object to a reference.

Robotics & pose · molecular reconstruction · computer vision
all on band-limited data on the sphere.

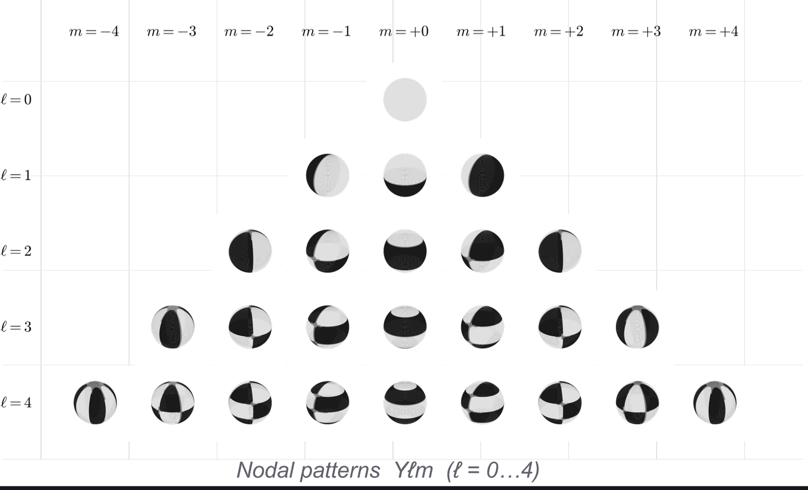
VOLUMETRIC: ✓ global & reliable ✗ but $O(L^4)$ — costly

LOCAL (ICP): ✓ cheap per step ✗ but needs init · local minima

WE WANT: deterministic · global · fast.

SPHERICAL HARMONICS

A Fourier transform wrapped around the sphere.


Nodal patterns $Y\ell m$ ($\ell = 0 \ldots 4$)

$$f(\theta,\phi) = \sum_{\ell=0}^{L} \sum_{m=-\ell}^{\ell} a_{\ell m}\, Y_{\ell m}(\theta,\phi)$$

$$Y_{\ell m} = N_{\ell m}\, P_\ell^m(\cos\theta)\, e^{im\phi}$$

ℓ — angular frequency; m — azimuthal order.

m → east–west nodal lines
ℓ-$|m|$ → north–south circles.
High $|m|$ modes hug the equator
the part of the boundary hologram reads best!

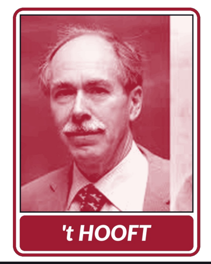
THE HOLOGRAPHIC IDEA

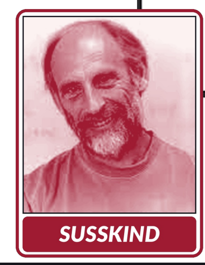
THE HOLOGRAPHIC PRINCIPLE

't Hooft (1993) & Susskind (1995):

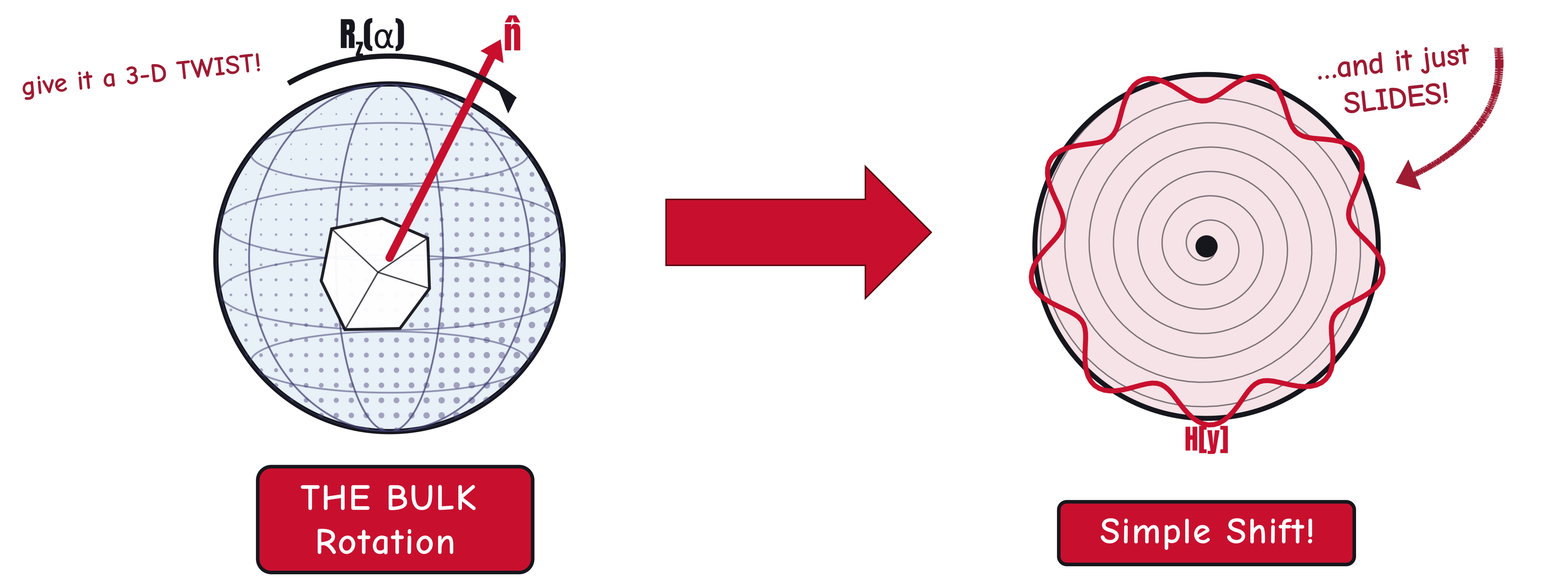
Given any closed surface, we can represent all that happens inside it by degrees of freedom on this surface itself.

HSA does the same by symmetry, not gravity.


'T HOOFT


SUSSKIND

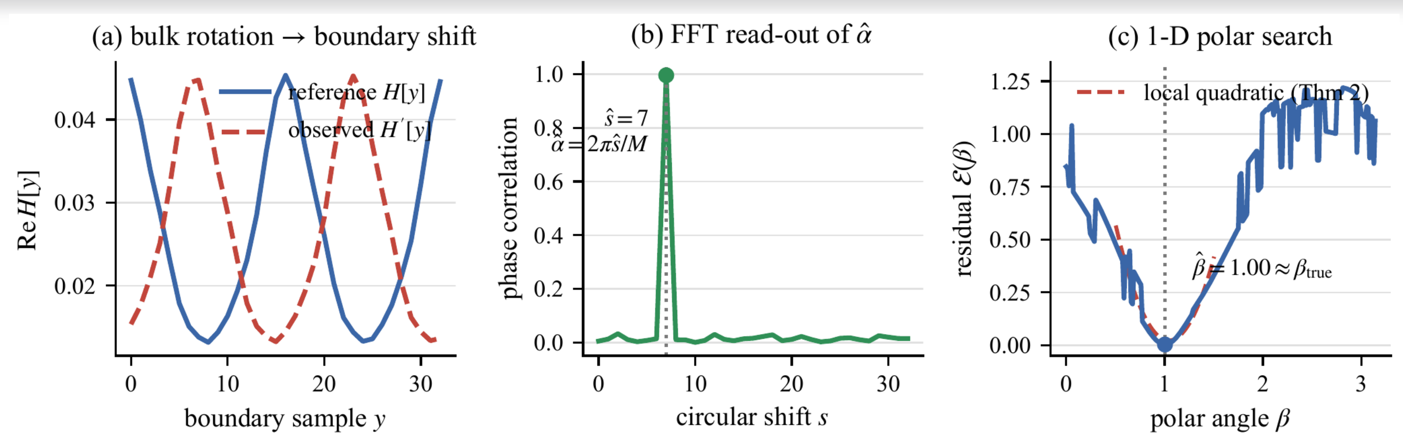
THE BULK-BOUNDARY MAP

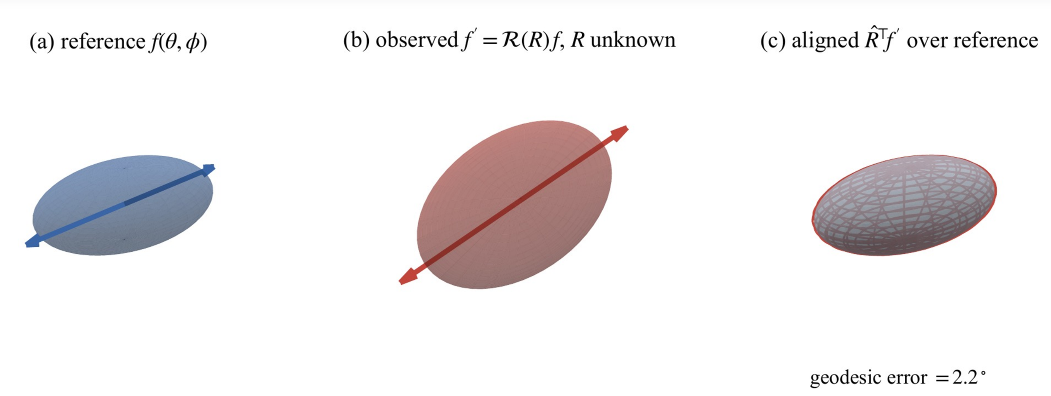


THE ALGORITHM

1	Gauge	$\mathbf{a}_\ell^{(g)} = D^\ell(G)\,\mathbf{a}_\ell$		quadrupole, β small
2	Holography	$H[y] = \sum_\ell \sum_{m=-\ell}^{\ell} w_\ell\, a_{\ell m}\, e^{i2\pi m y/M}$	$w_\ell = e^{-\tau\ell(\ell+1)}$	boundary hologram H[y]
3	Rotation → shift	$H'[y] \approx H\left[y - \frac{\alpha M}{2\pi}\right]$		Fourier shift on the boundary
4	Read α	$\hat{\alpha} = 2\pi\,\hat{s}/M$	$C[k] = \frac{\hat{H'}[k]\,\overline{\hat{H}[k]}}{\lvert\hat{H'}[k]\,\hat{H}[k]\rvert}$	FFT phase correlation
5	Polar search	$\mathcal{E}(\beta) = \sum_\ell \lVert d^\ell(\beta)^\dagger E_\ell(-\hat{\alpha})\,\mathbf{a'}_\ell - F_\ell(\hat{\gamma})\,\mathbf{a}_\ell \rVert^2$		1-D in β, γ per trial
6	Compose R	$\hat{R} = G_{\text{obs}}^{-1}\,R_z(\hat{\alpha})\,R_y(\hat{\beta})\,R_z(\hat{\gamma})\,G_{\text{ref}}$		undo the gauge

THE ALGORITHM IN ACTION





A RECIPROCAL PREDICTION

Analogy is a two way street. The boundary map is lossy (~4/L).
Pushing this purely geometric bound into black hole thermodynamics predicts a logarithmic area correction to the Bekenstein–Hawking entropy.

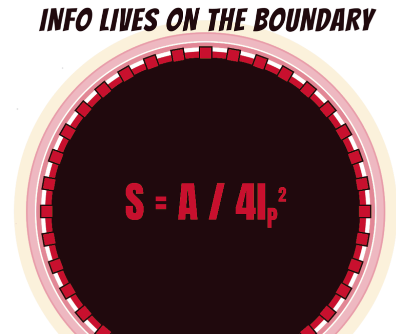
$$S = \frac{A}{4\ell_P^2} + c\log A + \cdots$$

$$\mathrm{E}\left[\frac{F_{\text{bnd}}^{(\lambda)}}{F_{\text{bulk}}^{(\lambda)}}\right] = \frac{n_\lambda^{\text{bnd}}}{n_\lambda^{\text{bulk}}} \le 1$$

$$\frac{F_{\text{bnd}}}{F_{\text{bulk}}} = \frac{4}{L} - \frac{6}{L^2} + \mathcal{O}(L^{-3})$$

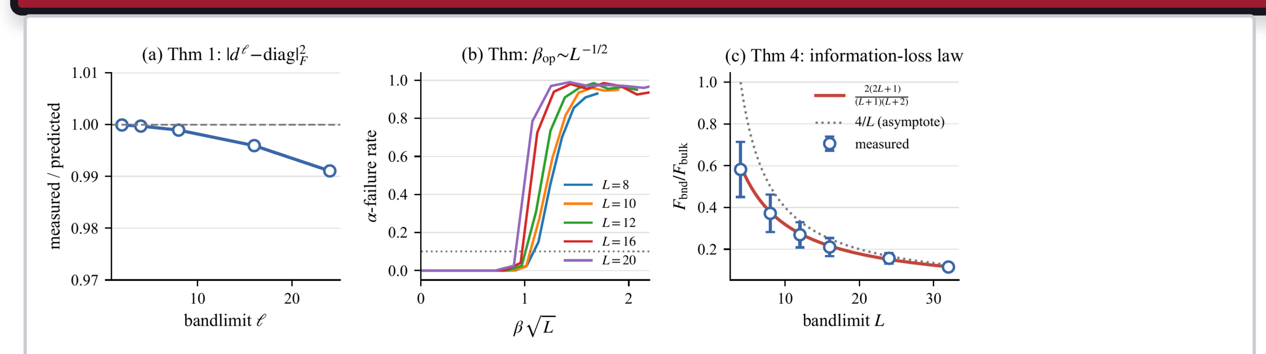
even BLACK HOLES agree!

INFO LIVES ON THE BOUNDARY

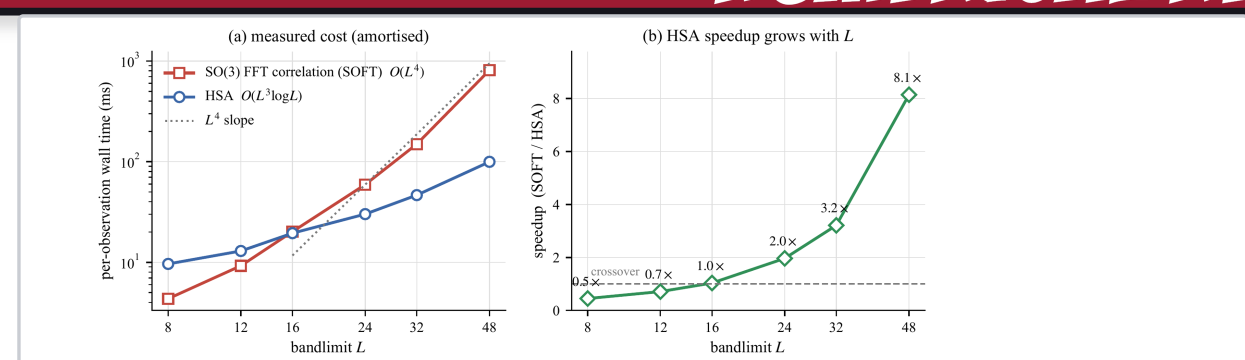


Schur's lemma + Cauchy–Schwarz set c from the bulk symmetry group
Same structure without gravity!

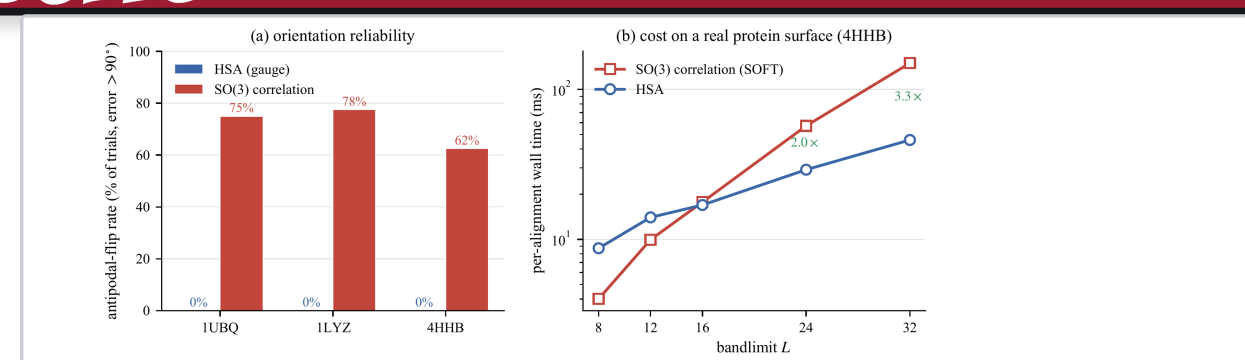
NUMERICAL RESULTS



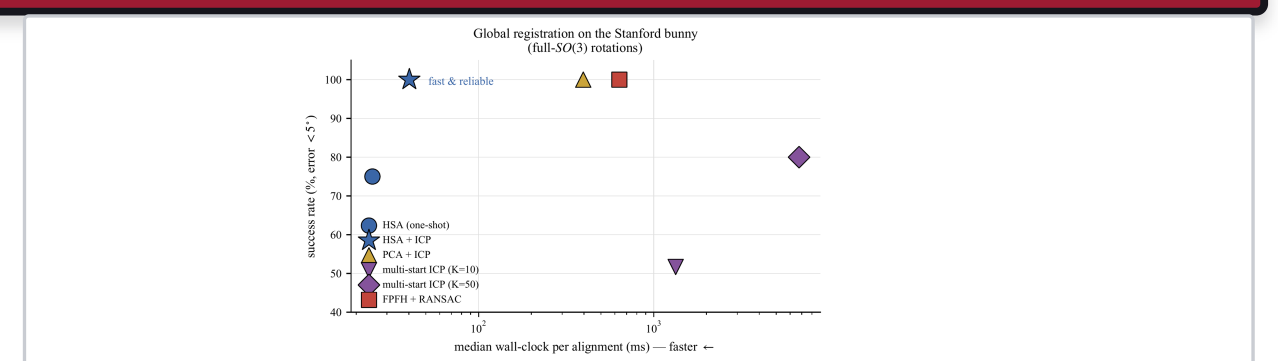
All four theorems verified numerically. Measured vs. predicted within ~1%.



HSA overtakes the O(L⁴) baseline at L=16; 8× faster by L=48 as the gap grows like L/log L.



On real proteins (PDB) antipodal flips drop 62–78% to 0%, staying 2–3× faster.



Stanford bunny: HSA+ICP is 100% reliable at 40 ms, the fast and reliable corner, ~16× under FPFH+RANSAC.

References: Kostelec–Rockmore '08 · Driscoll–Healy '94 · Kuglin–Hines '75 · Besl–McKay '92 · Kovacs–Wriggers '02 · 't Hooft '93 · Almheiri–Dong–Harlow '15 · Cover–Thomas '06 · Cohen et al. '18.